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# Secondary Dynamic Coordinated Optimization of Tooth Profile for the Cycloid Gear in RV Reducers under Multi-factor Coupling

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## Abstract:

*The cycloid gear modification affects the meshing quality and transmission accuracy of RV speed reducers. However, there are few effective methods to find the optimal modification value. This paper adopts a secondary dynamic coordinated optimization method to obtain the optimal modification value. Firstly, by analyzing the meshing characteristics of the cycloid gear in RV speed reducers, the various factors which influence the RV speed reducers, transmission characteristics was established, and then the design variables were extracted. Secondly, on the constraints that transmission accuracy, contact stress, transmission smoothness and other transmission characteristics be satisfied, the maximum transmission torque was taken as the optimization objective and the meshing range of the cycloid gear was dynamically selected. Consequently, the maximum transmission torque and corresponding design variables can be obtained utilizing optimal algorithm. Finally, the meshing number and position of the cycloid gear can be adjusted, and the optimal solution satisfying all the constraints can be obtained after a second optimization. It is found that the cycloid gear profile modification obtained in this way can increase the rated load torque of RV reducers compared to existing products, and the transmission accuracy can also be enhanced. Consequently, a new solution can be provided for the profile modification of cycloid gears.*

## Keywords:

*RV speed reducer, Cycloid gear, Modification, Optimization*

## 1 INTRODUCTION

RV speed reducers, stemming from cycloid reducer, is a new kind of planetary gear transmission mechanism. Since 1986, it has become one of the main parts of industrial robots due to its large transmission ratio, high transmission efficiency and precision, small backlash, large stiffness, compact structure and so forth. Cycloid gear and pin gears are the main components that affect the transmission accuracy, backlash and stiffness in RV reducers. Ideally, the meshing points between cycloid gear and pin gears are half that of the pin gears. Whereas, owing to some machining and assembly errors as well as lubrication requirements, the tooth profile of cycloid gear will be modified to a certain value. Therefore, the meshing points will be less than half the pin gears. After reasonable modification, the torsional stiffness of the cycloid gear can be effectively increased to reduce the torsional deformation at rated load, and the transmission precision can be improved. Meanwhile, the modification can make the deformation at each meshing point more balanced and



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reduce the contact stress to improve the load and anti-impact capability.

Extensive researches on the profile modification technology of cycloid gears have been conducted by many researchers. Reference [1] proposed that the optimal modified result should make the clearance between the new tooth profile and the original tooth profile generated by equidistant and shift modification as small as possible on the premise of maintaining specific radial clearance. So there are four kinds of combined modification methods. The meshing stiffness of cycloid gear are various with different combined modification methods. And the positive equidistant plus negative radial-moving modification method can obtain the maximum meshing stiffness when the combined modification produced the maximum initial clearance of the same size [2]. The tooth profile of the cycloid gear is a smooth curve which can be segmented into several curves by analyzing the pressure angle at each position of cycloid gear tooth profile. The most efficient range is taken as the working section tooth profile, and then the modification value can be obtained by further calculation [3]. Reference [4] realized the control of contact force and contact stress of cycloid gear by adjusting the meshing clearance of five key points in the gear meshing section of cycloid gear after modification. In order to reduce the computation of finite element method, the mathematical method is used to calculate the force of each meshing point after the cycloid gear was modified. Then the deformation and contact stress of the cycloid gear can be analyzed more quickly and accurately after applying the corresponding loads of the meshing points in the finite element model [5]. The cycloid reducers contain many components which could affect the transmission characteristic of the reducers. So the influence of eccentric distance and the radius of pin gear on the tooth profile of cycloid gear are analyzed in reference [6]. During the transmission process, the meshing stiffness and torsional stiffness are changing with the time, reference [7] analyzed the influence of tooth profile modification and eccentric error on the time-varying meshing stiffness of cycloid gear. And it proved that the variation characteristics of meshing stiffness and torsional stiffness with time after modification. Because of the limitation of the Hertz theory, reference [8] demonstrated the influence of profile modification on stress distribution of cycloid gear meshing point by applying non-Hertz theory. In the field of dynamics, reference [9-10] analyzed the meshing characteristics during the cycloid gear transmission process by using dynamic model, providing a new constraint for cycloid gear tooth profile's modification.

The existing methods tend to treat the influence of various factors on the meshing performance separately and ignore the relationship between them. During the process of determining the modification value, the initial modification value is usually determined in advance. Then, the initial value is iteratively calculated until the reducer meets a specific performance requirement. For instance, if only the stress homogenization is pursued to improve stress condition, the influence between meshing range and efficiency will not be considered sufficiently, so the efficiency may not reach the required standards. Therefore, the modification value determined by this kind of method does not take the coupling effect of multi-factors into account. Actually, the RV speed reducer products are to be qualified as long as the performance index including transmission accuracy, torsional stiffness, load torque and other performance indicators exist within the allowable range. If a certain performance is pursued excessively, some other performances will be too low and unnecessary waste will appear. Thus, an appropriate and general method should be applied to calculate the optimal modification value so the results after modification can realize the global optimum on the premise of satisfying all requirements of a RV reducer.

Taking the influence of multi-factor coupling effect on the RV reducers into consideration, this paper establishes a coupled mathematical model of the meshing performance, influence factors and modification value to analyze the interaction between them; thereafter, in order to improve the load capacity and anti-impact capability of the RV speed reducers, the maximum torque of the cycloid gear after profile modification is taken as the objective function. Because the transmission torque is a function of torsional stiffness and torsional deformation, the contact stress, transmission stability, initial clearance and rated torque should also be satisfied while meeting the torsional stiffness and torsional deformation requirements. Therefore, this paper

takes the coupling influence factors as the constraints. Then, the modification of cycloid gear with the maximum transmission torque is calculated by genetic algorithm when all the constraint conditions are satisfied. At last, it is determined whether to adjust the relevant constraints to carry out the second dynamic coordination optimization according to the condition of the meshing range after the first optimization. Thus, the optimum profile modification value can be obtained, which can make the meshing performance of a speed reducer reach the optimum.

## 2 CHARACTERISTIC PARAMETERS

### 2.1 Geometric parameters of cycloid gear

In the process of transmitting torque, there is an elastic deformation on the meshing points between cycloid gear and pin gears, which leads to a transmission error. The cycloid gear is regarded as an ideal rigid body, and the relative displacement caused by elastic deformation between the cycloid gear and pin gear is equivalent to the movement of the pin gear, as shown in Figure 1.

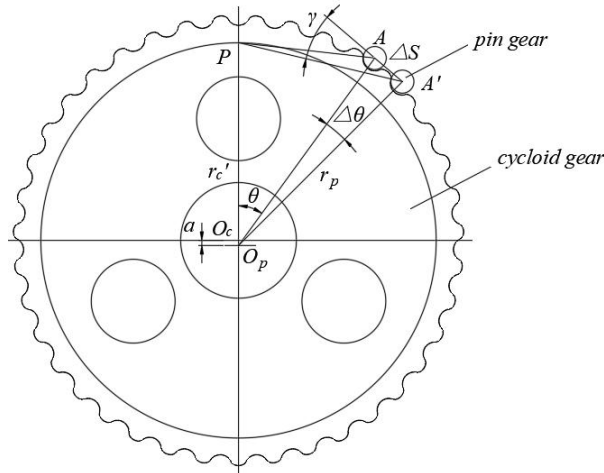


Figure 1: Transmission error diagram

where  $\theta$  is the phase angle of the meshing point.  $a$  is the eccentric distance.  $r_c'$  is the distance from the pitch point  $P$  to the cycloid gear center  $O_c$ .  $r_p$  is the distance from pin center to  $O_p$ .  $\Delta\theta$  is the angular deformation.  $A$  is the initial position of the pin center.  $A'$  is the position of the pin center after angular deformation.  $\Delta S$  is the circumferential displacement of the pin center.  $\Delta S_p$  is the equivalent displacement of the pin center along the normal direction  $PA'$ , i.e.  $\Delta S_p = \Delta S \cdot \cos \gamma$ . The geometric relationship is as follows:

$$PA = \sqrt{(r_c' + a)^2 + r_p^2 - 2(r_c' + a)r_p \cos \theta}$$

$$PA' = \sqrt{(r_c' + a)^2 + r_p^2 - 2(r_c' + a)r_p \cos (\theta + \Delta\theta)}$$

$$\Delta S_p = r_p \cdot \frac{\pi}{180} \cdot \Delta\theta \left[ \begin{aligned} &\sin \frac{\Delta\theta}{2} \cdot \frac{r_p - (r_c' + a) \cos (\theta + \Delta\theta)}{\sqrt{(r_c' + a)^2 + r_p^2 - 2(r_c' + a)r_p \cos (\theta + \Delta\theta)}} \\ &+ \cos \frac{\Delta\theta}{2} \cdot \frac{(r_c' + a) \sin (\theta + \Delta\theta)}{\sqrt{(r_c' + a)^2 + r_p^2 - 2(r_c' + a)r_p \cos (\theta + \Delta\theta)}} \end{aligned} \right] \quad (1)$$

$$L = \frac{r_c' r_p \sin (\theta + \Delta\theta)}{\sqrt{(r_c' + a)^2 + r_p^2 - 2(r_c' + a)r_p \cos (\theta + \Delta\theta)}} \quad (2)$$

After the profile modification of the cycloid gear, there will be initial clearances between the cycloid gear and the pin gear in the other positions when one of the cycloid gear tooth meshes with a certain pin gear <sup>[11-12]</sup>.

The clearance  $\Delta(\theta)_i$  along the normal direction of the initial clearance PA' can be calculated by Eq.(3).

$$\Delta(\theta)_i = \Delta r_{rp} \left( 1 - \frac{\sin \theta_i}{\sqrt{1+K^2-2K\cos \theta_i}} \right) - \frac{\Delta r_p (1-K\cos \theta_i - \sqrt{1-K^2}\sin \theta_i)}{\sqrt{1+K^2-2K\cos \theta_i}} \quad (3)$$

Where K is the short width coefficient. Here,  $K = \frac{az_p}{r_p}$ .

## 2.2 Meshing stiffness of cycloid gear

During the meshing process of cycloid gear, the elastic deformation size can be approximately calculated by Hertz formula [13-14]. The extrusion deformation value of the pin gear can be calculated by Eq.(4).

$$t_z = \frac{4F_i \rho_c (1-\mu^2)}{\pi b E r_{rp}} \quad (4)$$

The extrusion deformation value of the cycloid gear tooth can be calculated by Eq.(5).

$$t_c = \frac{4F_i \rho_i (1-\mu^2)}{\pi b E \rho_i} \quad (5)$$

Where  $F_i$  is the force at the  $i$ -th meshing point.  $E$  is the elastic modulus.  $\mu$  is the Poisson ratio.  $b$  is the thickness of the cycloid gear.  $r_{rp}$  is the radius of the pin gear.  $\rho_i$  is the curvature radius of the cycloid gear tooth profile at the  $i$ -th meshing point.  $\rho_c$  is the equivalent curvature radius at the  $i$ -th meshing point.  $\rho_c$  can be calculated by Eq.(6-7).

$$\rho_c = \frac{\rho_i r_{rp}}{\rho_i - r_{rp}} \quad (6)$$

$$\rho_i = \frac{(r_p + \Delta r_p)(1+K'^2-2K'\cos \theta)^{\frac{3}{2}}}{K'(z_p+1)\cos \theta - (1+z_p K'^2)} + (r_{rp} + \Delta r_{rp}) \quad (7)$$

Because the elastic deformation of the meshing point is very small, so this paper considers the relationship between the elastic deformation and the contact force as a linear relation, which can meet the accuracy requirement [3]. The meshing stiffness of the pin gear can be calculated by Eq.(8).

$$k_z = \frac{F_i}{t_z} = \frac{\pi b E r_{rp}}{4 \rho_c (1-\mu^2)} \quad (8)$$

The meshing stiffness of the cycloid gear can be calculated by Eq.(9).

$$k_c = \frac{F_i}{t_c} = \frac{\pi b E |\rho_i|}{4 \rho_c (1-\mu^2)} \quad (9)$$

According to Eq.(6-7), it can be deduced that the meshing stiffness of a single pair of gear teeth is as follows.

$$k = \frac{k_z k_c}{k_z + k_c} = \frac{\pi b E |\rho_i| (\rho_i - r_{rp})}{4 \rho_i (1-\mu^2) (r_{rp} + |\rho_i|)} \quad (10)$$

The relationship between the meshing stiffness of a single pair of teeth and the curvature radius of the cycloid gear tooth profile is shown in Figure 2.

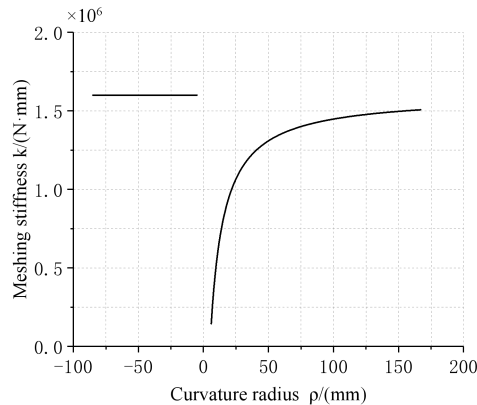


Figure 2: Variation of the meshing stiffness with curvature radius

When the cycloid gear profile is convex, the curvature radius is negative and the contact stiffness is a fixed value, which is independent of the curvature radius. When the tooth profile of cycloid gear is concave, the curvature radius is positive, and the contact stiffness is proportional to the curvature radius.

### 3 MULTI-FACTORS COUPLING MODEL

For a specific type of RV reducers, the transmission accuracy is determined by the torsional stiffness of cycloid gear when the load torque is constant during the transmission of torque. According to the equation of torsional stiffness [9]:

$$C = \sum C_j = \sum k_j \cdot L_j^2 \quad (11)$$

Where  $j$  is the meshing point.  $C_i$  is the torsional stiffness at the  $i$ -th meshing point.  $k_j$  is the meshing stiffness at the  $i$ -th meshing point.  $L_j$  is the arm of force at the  $i$ -th meshing point. The quantity and position of the meshing points change with the meshing range. And the corresponding meshing stiffness and arm of force will be quite different, which means the torsional stiffness generated at each meshing point is different. Therefore, the total torsional stiffness are determined by the meshing range.

The equation of the transmission accuracy is

$$\Delta\theta = \frac{\tau}{C} \quad (12)$$

The transmission accuracy is determined by torsional stiffness when the load torque is constant. As shown in Figure 3, there is a maximum torsional stiffness when the meshing range is within the peak and its adjacent interval of the torsional stiffness at a single meshing point.

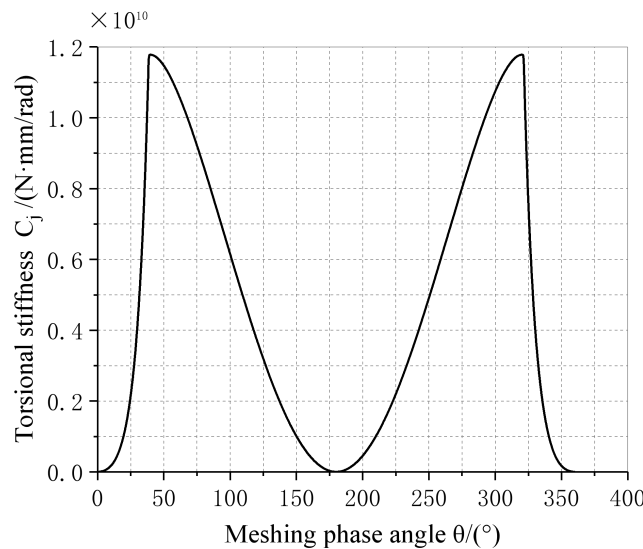


Figure 3: Variation of the torsional stiffness at a single meshing point

According to the Eq.(1), the torsional deformation is a function of equivalent deformation of the pin gear at the meshing point and the position of the meshing point. The relationship between them is shown in Figure 4.

After the profile modification of the cycloid gear, there will be initial clearance between the cycloid gear and the pin gears during the transmission. When the equivalent deformation is greater than the initial clearance, the cycloid gear is meshed with the pin gear, which results in contact deformation. Otherwise, the meshing does not occur. As shown in Figure 5, the intersection area between the equivalent deformation curve and the initial clearance curve is the meshing range in which contact deformation occurs. And the difference of the vertical coordinate values is the contact deformation at each meshing point.

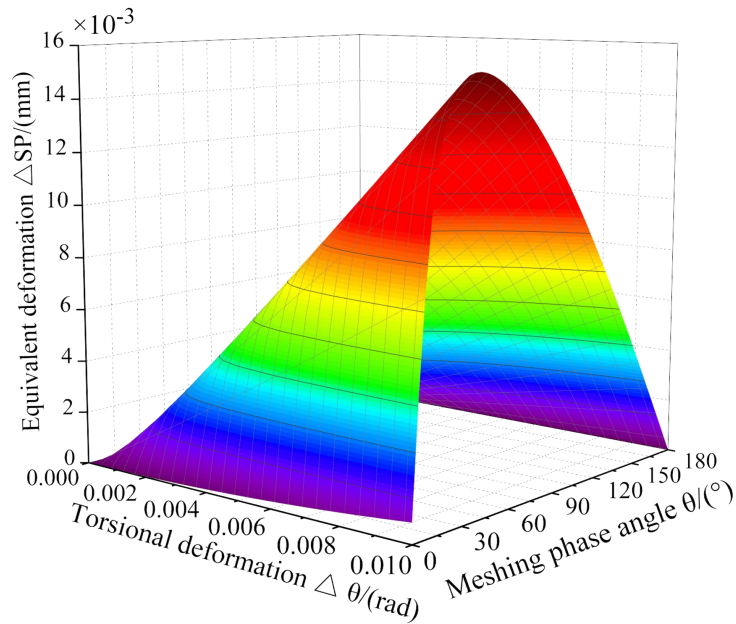


Figure 4: Variation of the torsional deformation with the equivalent deformation and the phase angle of meshing

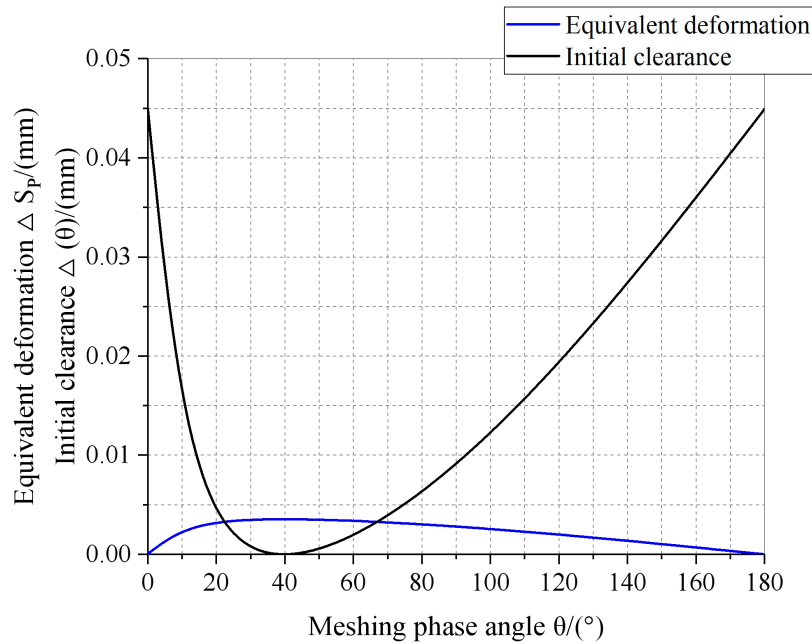


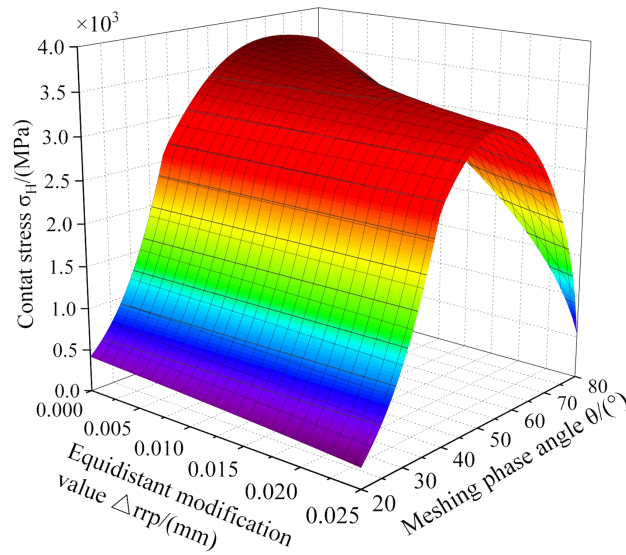
Figure 5: Variation of the equivalent deformation and initial clearance with phase angle of meshing

The contact stress between the cycloid gear and pin gear can be calculated by Eq.(13).

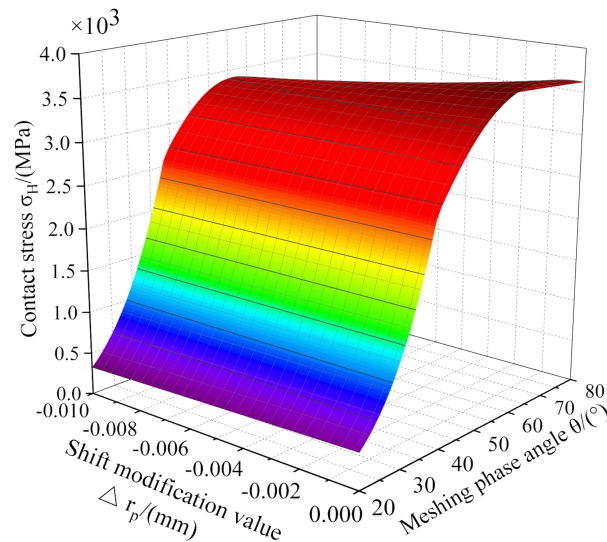
$$\sigma_H = 0.418 \sqrt{\frac{EF_i}{b\rho_c}} = 0.418 \sqrt{\frac{Ek_i(\Delta S_{pi} - \Delta(\theta)_i)}{b\rho_c}} \quad (13)$$

The contact stress of cycloid gear is determined by contact deformation, meshing stiffness and composite curvature radius of meshing point. The smaller the contact stress value is, the greater the contact force that can be borne by the meshing point, which means the greater the torque can be transmitted. However, the transmission accuracy will also change with the transmitted torque. As shown in Figure 6, the overall performance of cycloid gear can be adjusted by changing the profile modification value of the cycloid gear.





(a) Variation of contact stress with equidistant modification



(b) Variation of contact stress with shift modification

Figure 6: Variation of contact stress with profile modification

In summary, the change of profile modification of the cycloid gear will lead to the change of initial clearance and curvature radius. The meshing range is determined by the values of the initial clearance and equivalent deformation. The meshing stiffness is determined by the meshing range and the curvature radius at the meshing point. Thus, the torsional stiffness of cycloid gear can be determined. And the transmission accuracy under different load torque is finally controlled. Meanwhile, the equivalent deformation of cycloid gear is determined by the transmission accuracy and the meshing points. And the equivalent deformation, initial clearance and curvature radius of each meshing point determine the contact stress at each meshing point. The smaller the contact stress, the more torque the cycloid gear can transmit, which causes the change of the transmission accuracy. The coupling relationship between the performances of cycloid gear is shown in Figure 7.

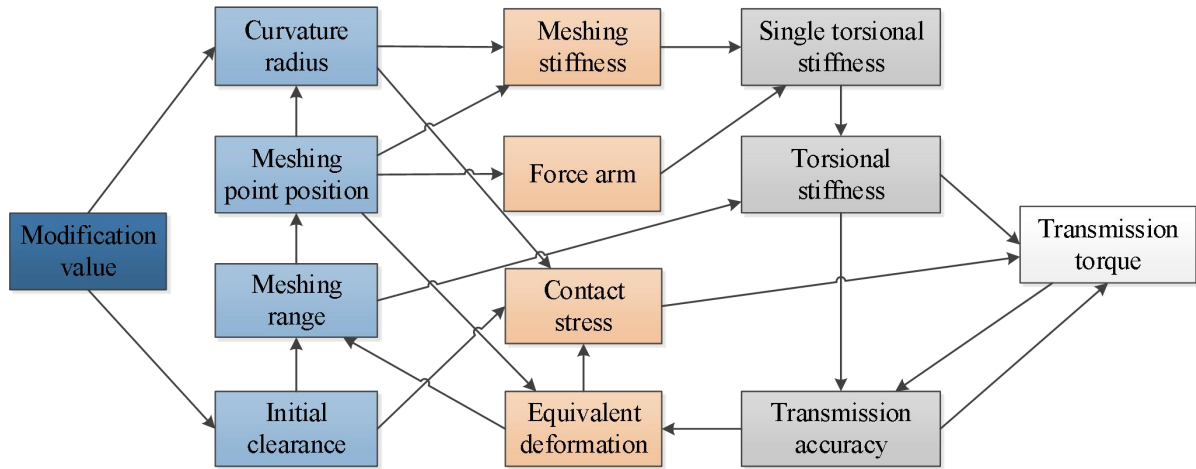


Figure 7: Coupling relationship between the performances of cycloid gear

## 4 DYNAMIC OPTIMAL DESIGN FOR THE CYCLOID GEAR

In this paper, the tooth profile modification of the existing RV reducers is optimized and its structure size is not changed. The tooth profile modification of cycloid gear is usually divided into three method: equidistant modification, radial-moving modification and rotated angle modification, which represented by  $\Delta r_p$ ,  $\Delta r_p$  and  $\Delta \delta$  respectively.

### 4.1 Design variables

Because the rotated angle modification is more complicated, it will greatly increase the grinding time, so the combination of equidistant modification and radial-moving modification is often adopted <sup>[1]</sup>. As shown in Figure 7, the value of transmission torque is determined by torsional stiffness and transmission accuracy  $\Delta \theta$ . And torsional stiffness is a function of the modification. Therefore, the design variables of the whole optimization are  $\Delta \theta$ ,  $\Delta r_{rp}$  and  $\Delta r_p$ .

### 4.2 Objective function

In the performance parameters of RV reducers, transmission accuracy, stiffness, reliability, lubrication and other performances are the requirements that must be satisfied. At the same time, greater transmission torque means higher load capacity and anti-impact capability. So the objective function is

$$T(\Delta \theta, \Delta r_{rp}, \Delta r_p)_{max} \quad (14)$$

### 4.3 Constraint conditions

As mentioned above, the transmission accuracy, contact stress, lubrication requirements, minimum curvature radius and rated load should be satisfied when optimizing the profile modification of cycloid gear. So the constraint conditions are as follows.

#### 4.3.1 Transmission accuracy constraint

In this paper, the transmission accuracy is the angle deviation caused by the elastic deformation between the cycloid gear and the pin gear. The maximum value of the transmission accuracy can be selected according to



its influence on the sensitivity of total transmission accuracy [15-16]. Therefore, the constraint condition is

$$0 < \Delta\theta < \eta\varphi \quad (15)$$

Where  $\eta$  is the proportional coefficient of influence on sensitivity.  $\varphi$  is the transmission accuracy.

#### 4.3.2 Contact stress constraint

The contact stress at meshing points of cycloid gear should not exceed allowable contact stress  $\sigma_{HP}$ . Because the cycloid gear and the pin gear are usually of the same material, the constraint condition of the contact stress is

$$\sigma \sqrt{\frac{EF_i}{b\rho_{cHPHmax}}} \quad (16)$$

#### 4.3.3 Modification constraint

The minimum clearance between the root of the cycloid gear and the pin gear should be greater than the clearance  $\Delta_{min}$  which is used to generate oil film for lubrication. Therefore, the constraint condition is

$$\Delta r_{rp} + \Delta r_p \geq \Delta_{min} \quad (17)$$

Meanwhile, there are clearances between the cycloid gear and the pin gear in the other positions when one of the cycloid gear tooth meshing with a certain pin gear. As shown in Figure 5, because the initial clearance is a concave function, the second derivative is greater than 0. And when  $\theta = \arccos K$ ,  $\Delta(\varphi) = 0$ . Therefore, the constraint condition is

$$\frac{d^2\Delta(\theta)_i}{d\theta_i} = \Delta r_{rp} - \Delta r_p \frac{1}{\sqrt{1-K^2}} > 0 \quad (18)$$

#### 4.3.4 Minimum curvature radius constraint

During the transmission, the top cut and sharp corner must be avoided for the stationarity of the transmission. If the curvature radius of cycloid gear tooth profile is too small, it will lead to the top cut and sharp corner. As shown in Figure 8, the concavity and curvature radius of cycloid gear tooth profile will change with the meshing phase angle. In order to avoid the top cut or sharp corner, the curvature radius of cycloid gear tooth profile must satisfy the minimum curvature radius requirement [12]. Therefore, the constraint condition is

$$\rho_{i\text{concave}} \geq \frac{(1-K)^2}{z_p K - 1} r_p \quad (19)$$

$$|\rho_i|_{\text{convex}} \geq \begin{cases} r_p \sqrt{\frac{27(1-K^2)(z_p-1)}{(z_p+1)^3}}, & 1 > K > \frac{z_p-2}{2z_p-1} \\ \frac{(1+K)^2}{z_p K + 1} r_p, & \frac{z_p-2}{2z_p-1} \geq K \end{cases} \quad (20)$$

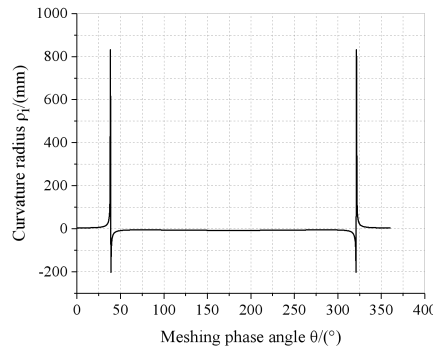


Figure 8: Variation of curvature radius with meshing phase angle

#### 4.3.5 Transmission torque constraint

According to reference [13], because the distribution of the force is not uniform during the transmission, the torque transferred by a single cycloid gear is

$$T_c = 0.55T \quad (21)$$

Where  $T$  is the rated load of the RV reducers. As shown in Figure 9, the torque applied to the center of the cycloid gear is generated by the force at the meshing points, which form the transmission torque.

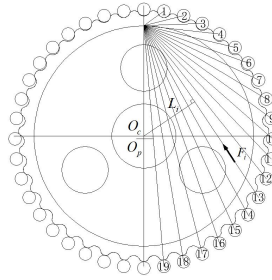


Figure 9: Torque distribution

The sum of the torque generated at the meshing points should be not less than the rated load torque, so the constraint condition is

$$T_c = \sum (k_i \cdot (\Delta S_{pi} - \Delta(\theta)_i) L_i) \geq 0.55T \quad (22)$$

Where  $k_i$  is the meshing stiffness of a single pair of teeth at the  $i$  – th meshing point.

#### 4.4 Optimization program

According to the meshing characteristic of cycloid gear, different modification value will cause different meshing range. If the meshing range is selected only according to the design experience, the calculation amount is too large and it is difficult to get the optimal solution. Therefore, the key of the optimum design is to realize the automatic selection and calculation of the meshing range by appropriate method. And the optimal value under the constraint conditions is obtained by calculation. This paper selected the meshing points automatically by computer programs. And then the genetic algorithm is used to optimize the calculation<sup>[17]</sup>. The calculation process is shown in Figure 10.

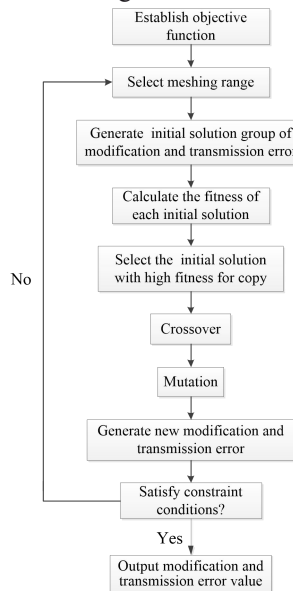


Figure 10: Flow diagram of optimization calculation

#### 4.5 Secondary optimization

During the transmission, the force arm is very short when the meshing phase angle  $\theta$  is too small, which will lead to the decrease of transmission efficiency. Meanwhile, when the meshing phase angle is too large, the relative sliding speed between the cycloid gear and pin gear will be too large, which leads to the increase of power loss and the decrease of efficiency. In addition, when the phase angle is too large, the curvature radius of cycloid gear is smaller, and the contact stress will increase, resulting in scuffing. The recommended range of meshing phase angle is  $25^\circ < \theta < 100^\circ$  [12]. Therefore, while optimizing the modification of cycloid gear, the meshing range of cycloid gear must be controlled.

According to reference [3], the adjustment of meshing range can be realized by changing the symbol of equidistant modification and radial-moving modification. Therefore, when the first optimization calculation is finished, the quantity and position of meshing points can be adjusted by increasing or decreasing the meshing range according to the distribution of the meshing range. As shown in Figure 11, the combination of positive equidistant and negative radial-moving modification can increase the meshing range compared with that of positive equidistant and positive radial-moving modification. Therefore, the meshing range can be changed through setting the value interval of equidistant and radial-moving modification. And the more suitable meshing range can be obtained.

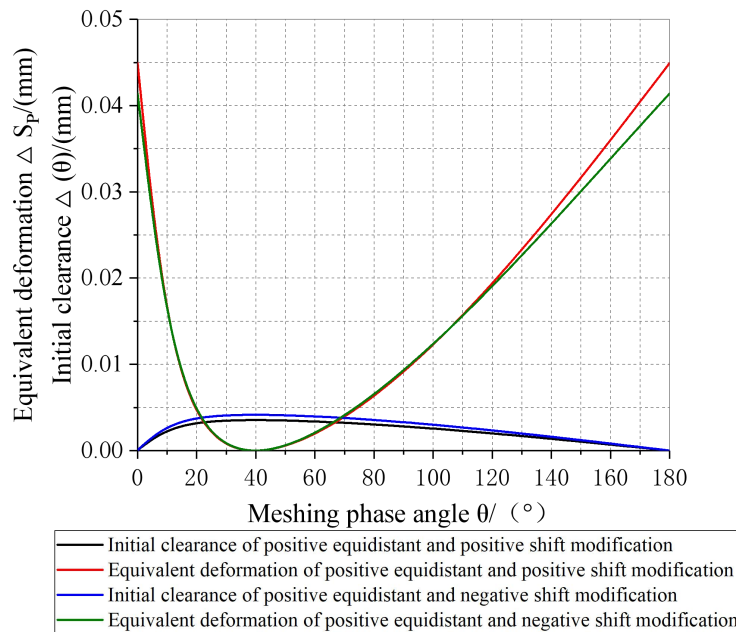


Figure 11: Variation of the meshing range with modification value

## 5 VERIFICATION OF OPTIMIZATION RESULT

In this paper, genetic algorithm is used to optimize profile modification of the cycloid gear in RV-40E reducers. As shown in Figure 12, because the interval of the solution satisfying the constraint condition is very small, the optimization calculation can become stable at a rapid pace. When the genetic algorithm calculation results evolve to the 13th generation, the optimal solution is obtained. Therefore, the maximum torque that can be transmitted by a single cycloid gear is  $275.3 \text{ N}\cdot\text{m}$ . The transmission accuracy  $\Delta\theta$  is  $0.01 \text{ rad}$ , the equidistant modification value  $\Delta r_{rp}$  is  $0.019 \text{ mm}$  and the radial-moving modification value  $\Delta r_p$  is  $-0.01 \text{ mm}$ .

The cycloid gear of RV-40E reducers parameters are shown in Table 1.

**Table 1 Cycloid gear of RV-40E reducers parameters**

Parameter	Value
Cycloid gear tooth number	39
Pin gears number	40
Eccentric distance (mm)	1.25
Cycloid gear thickness (mm)	9
Pin gear diameter (mm)	3

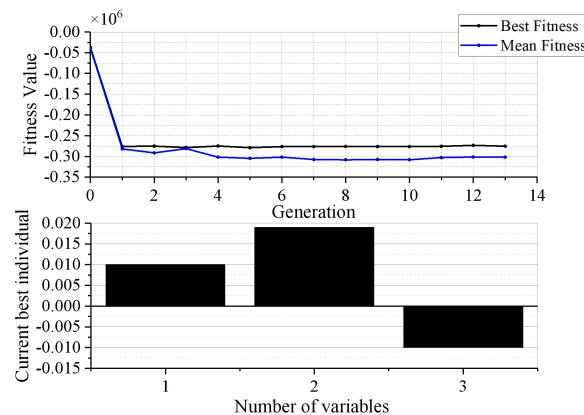


Figure 12: Optimization calculation result

### 5.1 Verification of multi-factors coupling optimization result by finite element analysis

According to the modification of cycloid gear after optimization, the model of cycloid gear is established by FEM. Because the cycloid gear is bolted between the planet carrier and rigid disk, so the cycloid gear can be regarded as a rigid body. Therefore, there is only a central hole on cycloid gear to apply equivalent torque for convenience of calculation. Then, the cycloid gear model is meshed with hexahedron elements. The contact element is established between the 1th to 19th pin gears and the corresponding cycloid gear tooth. The result of finite element analysis is shown in Figure 13.

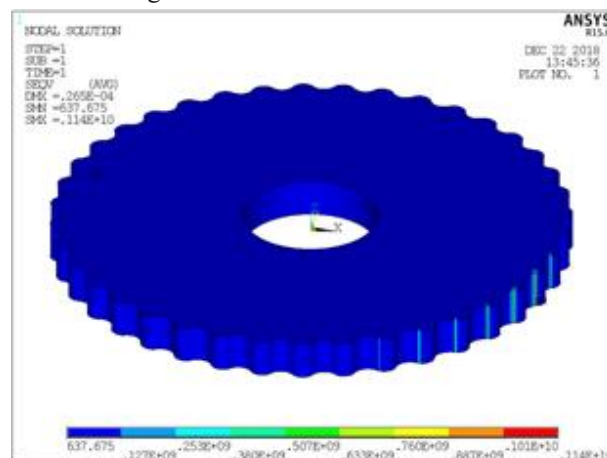


Figure 13: Finite element analysis result

The meshing points between cycloid gear and pin gear are 3th to 9th, which meets the requirements of

meshing range  $25^{\circ}$ - $100^{\circ}$ . The maximum contact deformation is 0.0265mm and the maximum stress is 1140MPa, which meets the design requirements.

### 5.2 Verification of single factor optimization result by finite element analysis

If only the stress homogenization is considered as the optimization objective, and multi-factors coupling is not considered, the finite element analysis result is shown in Figure 14.

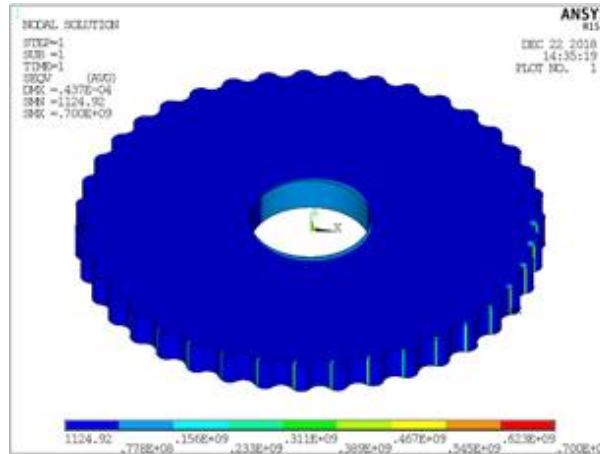


Figure 14: Finite element analysis result

The deformation of cycloid gear is 0.0437mm which is more than 0.0265mm after considering multi-factors coupling optimization, i.e. the transmission accuracy is lower. And the maximum contact stress is 700MPa which is less than 1140MPa after considering the multi-factors coupling optimization. But, according to the requirement of cycloid gear meshing range, the meshing points is form 2th to 16th, which is obviously not satisfied the requirement of meshing range  $25^{\circ}$ - $100^{\circ}$ .

In conclusion, considering the multi-factors coupling and the optimum modification determined by the optimization method with transmission torque as the optimization objective can meet the requirements of multiple performance indexes of cycloid gear simultaneously.

### 5.3 Experiment verification

In this experiment, the cycloid gears with the modification value obtained by multi-factors coupling optimization are machined, and the RV reducers with this cycloid gears are assembled. Then the torsional stiffness, transmission accuracy and transmission efficiency are compared between this RV reducers and that produced by a company. This test is carried out on the comprehensive performance test-bed of RV reducers. As shown in Figure 15, the test-bed can be used to detect the efficiency, torsional stiffness, backlash, transmission error and startup torque of the RV reducer within 5000N.m. Meanwhile, the temperature, vibration and noise of the reducer can be detected in real time. The double-range torque sensor (200N.m/20N.m) is adopted on the input terminal. The double-range torque sensor (5000N.m/500N.m) is adopted on the output terminal. And the drive motor is controlled by Siemens servo motor. The measurement accuracy of torque is  $\pm 0.1\%$ FS, the measurement accuracy of rotational speed is  $\pm 1$ r/min. The fastest update speed of angle measurement is 1ms, and the speed of data recording is 100ms each time. The input terminal adopts the rotary encoder with the measurement accuracy of  $\pm 1.5''$ , and the output terminal adopts the rotary encoder with the measurement accuracy of  $\pm 2.38''$ , which ensures the measurement accuracy of the angle error.

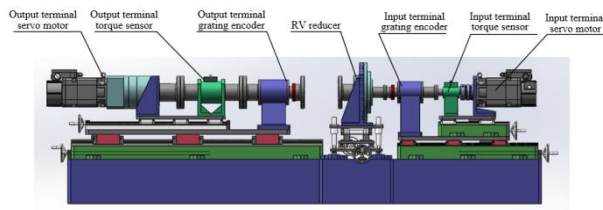
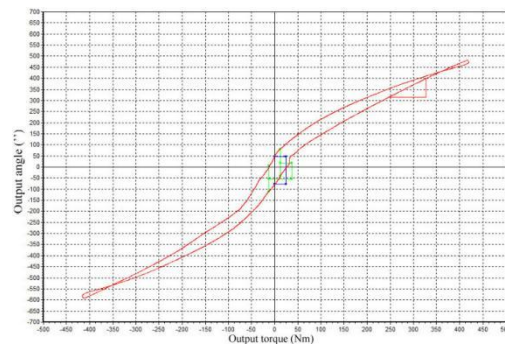


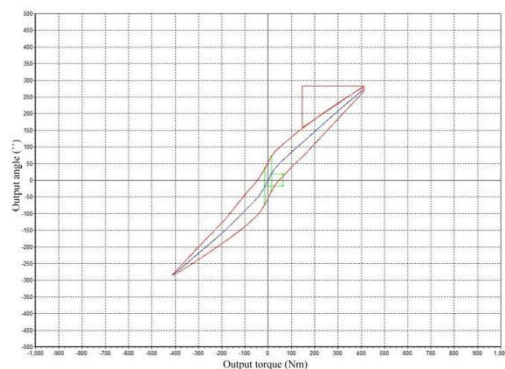
Figure 15: Comprehensive performance test-bed of RV reducers

### 5.3.1 Torsional stiffness experiment

This experiment is used to detect the torsional stiffness of cycloid gear transmission, that is, the torsional stiffness of output terminal. So the input terminal is fixed by the fixture of the test-bed in order to eliminate the influence of planetary gear part on the experiment result. During the detection, the servo motor and reducer on the output terminal of the test-bed applied torque to the RV reducers, and the torque was gradually increased from 0 N·m to the rated torque. And then the torque was applied in the opposite direction, which gradually loaded from 0 N·m to the rated torque. During the experiment, the driving torque and angle variation are monitored in real time by the torque sensor and rotary encoder of the output terminal. And the data is recorded every 0.1s. The experiment results are shown in Figure 16.



(a) Torsional stiffness experiment result of a RV-40E reducer produced by a company



(b) Torsional stiffness experiment result of a RV-40E reducer after optimization

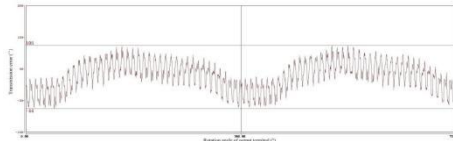
Figure 16: Torsional stiffness experiment results of a RV-40E reducer

The experiment results can be directly obtained from the software interface. Because the stiffness experiment is to apply torque in clockwise and counterclockwise directions, the backlash of the RV reducers is also measured at the same time. It is shown that the torsional stiffness of the optimized RV reducers is significantly increased and the backlash is reduced, which proves that this optimization method in improving torsional stiffness and reducing the backlash of the RV reducers is feasible.

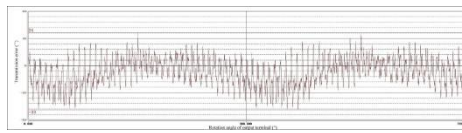


### 5.3.2 Transmission accuracy experiment

When measuring the transmission accuracy, the output terminal applied a constant load of  $50\text{N}\cdot\text{m}$ , and the input terminal applied torque and rotational speed. The rotary encoder was used to monitor the rotation angle of the input terminal and output terminal in real time. And the data is recorded every  $0.1\text{s}$ . Then the rotation angle of the input terminal is divided by the transmission ratio, and the difference between this result and the rotation angle of the output terminal is the real-time transmission accuracy. The experiment results are shown in Figure 17.



(a) Transmission accuracy experiment result of a RV-40E reducer produced by a company



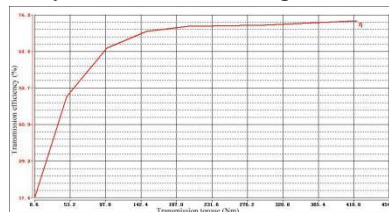
(b) Transmission accuracy experiment result of a RV-40E reducer after optimization

Figure 17: Transmission accuracy experiment results of a RV-40E reducer

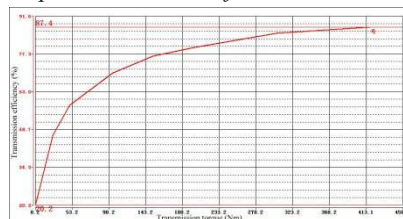
The results indicate that the total transmission accuracy presents a periodic change, which is mainly caused by the periodic change of the meshing stiffness during the transmission of the RV reducers. And the variation of local transmission accuracy is mainly caused by the vibration of reducer and the change of meshing clearance. Under the load of  $50\text{N}\cdot\text{m}$ , the transmission accuracy of the optimized RV reducers is significantly improved, which is in the interval of  $-33''$  to  $24''$ . Therefore, the transmission accuracy of the optimized RV reducer can satisfy the accuracy requirement.

### 5.3.3 Transmission efficiency experiment

During the detection of transmission efficiency, torque and constant speed are applied at the input and output terminals respectively. When the torque is gradually increasing to the rated torque, the data of input and output torque and speed are collected in real time. The data is recorded every  $0.1\text{s}$ , and the real-time transmission efficiency can be obtained by calculation. The experiment results are shown in Figure 18.



(a) Transmission efficiency experiment result of a RV-40E reducer produced by a company



(b) Transmission efficiency experiment result of a RV-40E reducer after optimization

Figure 18: Transmission efficiency experiment results of a RV-40E reducer

It is concluded that the transmission efficiency is proportional to the load torque, and the transmission efficiency is highest when the load torque is maximum. The transmission efficiency of the optimized RV reducers has been improved, strongly proving the feasibility of the optimization method in improving the transmission efficiency in the RV reducers.

## 6 CONCLUSIONS

(1) Since the influence of the cycloid gear modification value on the meshing performance for RV reducers suffers from coupling effect of multi-factors, the coupling mathematical model among meshing performance, influence factors and modification value is established, which provides theoretical support for the establishment of mathematical model for optimization.

(2) The genetic algorithm is used to optimize the mathematical model of cycloid gear in each meshing range. Then, the optimum modification value and maximum transmission torque of cycloid gear are obtained after the first optimization.

(3) A secondary dynamic optimization method is proposed in this paper. On the basis of the first optimization result and the corresponding meshing condition of the cycloid gear, this method properly adjusts the value range of cycloid gear modification properly. Then a more suitable meshing range and the corresponding optimum modification value are obtained.

(4) According to the experiment results, the proposed secondary dynamic optimization method considering multi-factors coupling effects in this paper can obtain the optimal profile modification value for the cycloid gear. The requirements for multiple meshing performance can be satisfied at the same time using this method.

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